ABHINAV ACADEMY
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Mathematics By
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(MATHEMATICS)

Level-2, TEST-COORDINATE SYSTEM AND COORDINATES

Name: _____ **Mob No.** _____

1. If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is _____.

2. Let an ellipse with centre (1,0) and latus rectum of length $\frac{1}{2}$ have its major axis along $x-$axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to_____.

3. The line $x = 8$ is the directrix of the ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the $x-$axis at Q, then $(3PQ)^2$ is equal to _____.

4. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If $(1, \alpha)$ lies on C, then $10\,\alpha^2$ is equal to _____.

5. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is_____.

6. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to _____.

7. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3. If the points (x, y) satisfying $A^2 + xA + y\,I = O$ lie on a hyperbola, whose transverse axis is parallel to the $x-$axis, eccentricity is e and the length of the latus rectum is l, then $e^4 + l^4$ is equal to _____

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Rough Work

8. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x = \pm \dfrac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _____.

9. Let S be the focus of the hyperbola $\dfrac{x^2}{3} - \dfrac{y^2}{5} = 1$, on the positive $x-$axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S. If O is the origin and SAB is a diameter of C, then the square of the area of the triangle OSB is equal to_____.

10. Let the latus rectum of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{b^2} = 1$ subtend an angle of $\dfrac{\pi}{3}$ at the centre of the hyperboal. If b^2 is equal to $\dfrac{1}{m}(1+\sqrt{n})$, where l and m are co-prime numbers, then $l^2 + m^2 + n^2$ is equal to_____.

11. Let the foci and length of the latus rectum of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b$ be $(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\dfrac{x^2}{b^2} - \dfrac{y^2}{a^2 b^2} = 1$ equals

12. Let the eccentricity of an ellipse $\dfrac{x^2}{b^2} - \dfrac{y^2}{a^2 b^2}=1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is_____.

13. Let $H_n: \dfrac{x^2}{1+n} - \dfrac{y^2}{3+n} = 1, n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21l$ is equal to_____.